



Semester One Examination, 2025

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

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In words

Circle your Teacher's Name: Ms Lee Mr Liu Mr Mohammadi Mrs Murray

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	91	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (91 Marks)

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8

(3 marks)

Solve the following equation

$${}^xP_2 + {}^xP_3 = 3x^2 + 7x$$

Solution
${}^xP_2 + {}^xP_3 = 3x^2 + 7x$ $\frac{x!}{(x-2)!} + \frac{x!}{(x-3)!} = 3x^2 + 7x$ $x(x-1) + x(x-1)(x-2) = 3x^2 + 7x$ $x^3 - 2x^2 + x = 3x^2 + 7x$ $x = \cancel{1}, \cancel{0}, 6$ $\therefore x = 6$
Specific behaviours
✓ substitutes into permutations formula ✓ simplifies factorials correctly ✓ solves correctly, rejecting solutions ≤ 0

Question 9

ROUNDING QUESTION

(10 marks)

Consider the vectors $\vec{a} = \begin{pmatrix} 4\lambda + 3 \\ 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3\lambda - 4 \\ 1 \end{pmatrix}$, where λ is a constant.

(a) When $\lambda = -1$ determine

(i) the exact unit vector $\hat{r}$ when $\vec{r} = \vec{a} - \vec{b}$.

(2 marks)

Solution
$\vec{r} = \vec{a} - \vec{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -7 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ $\hat{r} = \frac{\sqrt{37}}{37} \begin{pmatrix} 6 \\ 1 \end{pmatrix}$
Specific behaviours
✓ correct vector difference ✓ correct unit vector, rationalised

(ii) the angle between $\vec{a}$ and $\vec{b}$ to the nearest 0.1 of a degree.

(2 marks)

Solution
Using CAS, $\text{angle} \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -7 \\ 1 \end{pmatrix} \right) = 55.3^\circ$.
Specific behaviours
✓ indicates use of CAS (teaching point) ✓ correctly rounded angle (answer only OK)

Alternate Solution
$\cos \theta = \frac{7 + 2}{\sqrt{5}\sqrt{50}}$ $\theta = 55.3^\circ$
Specific behaviours
✓ uses dot product ✓ correctly rounded angle (answer only OK)

(iii) the vector projection of $\vec{b}$ on $\vec{a}$.

(2 marks)

Solution
0 or 1 for a onto b? $\vec{r} = \frac{\begin{pmatrix} -7 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}}{\sqrt{5}\sqrt{5}} \times \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \frac{9}{5} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1.8 \\ 3.6 \end{pmatrix}$
Specific behaviours
✓ indicates correct method ✓ correct vector (accept either of last two steps, as fraction or decimal)

(b) Determine the exact value(s) of λ when $\vec{a}$ and $\vec{b}$ are

(i) parallel.

Solution
$\frac{4\lambda + 3}{2} = \frac{3\lambda - 4}{1} \Rightarrow \lambda = \frac{11}{2}$
Specific behaviours
✓ forms correct equation for parallel vectors ✓ correct value

Alternate Solution
If $\vec{a} \parallel \vec{b}$ then $\vec{a} = \mu \vec{b}$ $\begin{pmatrix} 4\lambda + 3 \\ 2 \end{pmatrix} = \mu \begin{pmatrix} 3\lambda - 4 \\ 1 \end{pmatrix}$ CAS solve $\begin{cases} 4\lambda + 3 = \mu(3\lambda - 4) \\ \mu = 2 \end{cases} \Rightarrow \lambda = \frac{11}{2}$
Specific behaviours
✓ forms correct equation for parallel vectors ✓ correct value

(ii) perpendicular.

(2 marks)

Solution
Allow $-0.\bar{6}$ as exact $\text{If } \vec{a} \perp \vec{b} \text{ then } \vec{a} \cdot \vec{b} = 0$ $\begin{pmatrix} 4\lambda + 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3\lambda - 4 \\ 1 \end{pmatrix} = 0$ $(4\lambda + 3)(3\lambda - 4) + (2)(1) = 0$ $12\lambda^2 - 7\lambda - 10 = 0$ $\lambda = -\frac{2}{3}, \lambda = \frac{5}{4}$
Specific behaviours
✓ indicates use of scalar product to form equation ✓ correct values

Question 10

(7 marks)

- (a) A teacher wrote eight distinct positive integers on the board. None of these numbers had a prime factor larger than 15.

- (i) Prove that there are two integers on the board that have the same prime factor. (2 marks)

Solution
Let the eight distinct positive integers be the pigeons, and the pigeonholes be the prime factors less than 15 i.e. 2, 3, 5, 7, 11, 13.
As there are six pigeonholes, and eight pigeons, then two integers have the same prime factor.
Specific behaviours
✓ identifies pigeons and pigeonholes ✓ uses pigeonhole principle to explain why two integers have the same factor

No penalty if they included 1 here as the logic stands (teaching point: remember 1 isn't prime).

- (ii) The teacher erased one of the eight integers on the board. Explain why it is now impossible to prove that there are two integers on the board that have the same prime factor. (2 marks)

Solution
There are now seven pigeons, and six pigeonholes. However, if one of the integers was 1, then this essentially creates a seventh 'pigeonhole'.
As there are seven pigeonholes, and seven pigeons, then it is possible that none of the numbers have the same prime factor.
Specific behaviours
✓ recognises that if '1' is included in the integers, it creates a 7th pigeonhole ✓ uses pigeonhole principle to explain why it is impossible to prove that two integers on the board have the same prime factor.

- (b) A group of 60 students was asked about their preferences for ice cream flavours. 28 liked chocolate, 22 liked vanilla and 25 liked strawberry. 5 liked all three flavours. 12 liked chocolate and vanilla, 9 liked strawberry and chocolate, and 8 liked vanilla and strawberry. **Clearly showing use of the inclusion-exclusion principle**, find the probability that a student chosen at random from this group liked only one flavour of ice cream. (3 marks)

Solution
Only likes choc = $ C - C \cap V - C \cap S + C \cap V \cap S = 28 - 12 - 9 + 5 = 12$
Only likes vanilla = $22 - 12 - 8 + 5 = 7$
Only likes strawberry = $25 - 9 - 8 + 5 = 13$
Probability of liking only one flavour = $\frac{12+7+13}{60} = \frac{32}{60} = \frac{8}{15} \approx 0.5333$
Specific behaviours
✓ demonstrates use of the inclusion-exclusion principle to find the number of students who only like chocolate ✓ finds the number of students who only like vanilla and only like strawberry ✓ correct probability as simplified fraction or decimal

Max 2 marks if they used a Venn diagram).

Any marks for 27/60?

Question 11

(9 marks)

- (a) The statement $\forall \alpha, \beta \in \mathbb{R}; \sin \alpha = \sin \beta \Rightarrow \alpha = \beta$ is written in the formal language of proof.

- (i) Write the statement in words as if you were reading it to an audience. (2 marks)

Solution	
For all real angles alpha and beta, if the sine of alpha equals the sine of beta then alpha equals beta.	
Specific behaviours	
✓ correctly describes set of angles (accept if symbols used)	
✓ correctly interprets conditional statement	

Accept "For all numbers a and b such that they are real numbers..."?

English is not a strong point!

- (ii) Briefly explain whether the statement is true or false. (1 mark)

Solution	
False: $\alpha = 0$ and $\beta = \pi$ is a counterexample.	
Specific behaviours	
✓ correct answer with justification	

- (b) Consider the following conditional statement about polygons with no crossed edges: If it is a quadrilateral, then the sum of its interior angles is 360° .

- (i) Write the contrapositive statement. (1 mark)

Solution	
If the sum of its interior angles is not 360° then it is not a quadrilateral.	
Specific behaviours	
✓ correct statement	

- (ii) Write the converse statement. (1 mark)

Solution	
If the sum of its interior angles is 360° then it is a quadrilateral.	
Specific behaviours	
✓ correct statement	

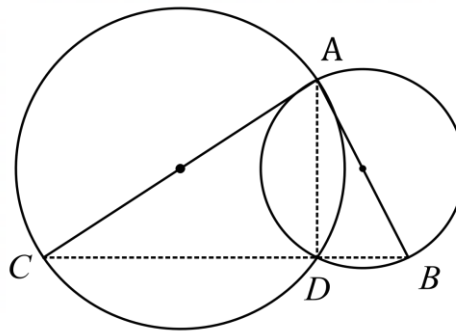
- (c) Let n be an integer. Prove that $\frac{n^3-n}{6}$ is always an integer. (3 marks)

Solution	
For $\frac{n^3-n}{6}$ to always be an integer, $n^3 - n$ must be divisible by 6. $n^3 - n = n(n^2 - 1) = n(n + 1)(n - 1)$ This is the product of three consecutive integers. Among any three consecutive integers, at least one must be even so their product is divisible by 2, and one must be a multiple of 3 so their product is divisible by 3. As the product is divisible by both 2 and 3, it is also divisible by 6 therefore $\frac{n^3-n}{6}$ will always be an integer.	
Specific behaviours	
✓ recognises that the numerator must be divisible by 6 ✓ factorises expression and recognises product of consecutive integers ✓ shows divisibility by 2 and 3 and concludes divisibility by 6	

Question 12

(6 marks)

Two circles intersect at points A and D so that their centres lie outside the other circle. The diameter AB of one circle is less than the diameter AC of the other circle, as shown in the diagram.



- (a) Prove that D lies on the straight line from B to C .

(3 marks)

Solution	
$\angle ADB = 90^\circ$ (AB is a diameter, D is angle in semicircle)	
$\angle ADC = 90^\circ$ (AC is a diameter, D is angle in semicircle)	
$\angle CDB = \angle ADB + \angle ADC = 90^\circ + 90^\circ = 180^\circ$	
Hence $\angle BDC$ is a straight angle and so D lies on the line BC .	
Specific behaviours	
✓ reasons that $\angle ADB$ is a right angle	
✓ reasons that $\angle ADC$ is a right angle	
✓ completes proof using straight angle	

Point E lies on minor arc AD of the larger circle and $\angle CAD = 62^\circ$.

- (b) Determine, with justification, the size of $\angle AED$.

(3 marks)

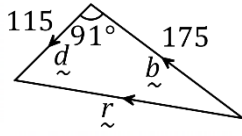
Solution	
	$\angle ACD = 90^\circ - 62^\circ = 28^\circ$ (complementary angles in right triangle ACD)
	$\angle AED = 180^\circ - 28^\circ = 152^\circ$ (opposite angles in cyclic quadrilateral $AEDC$)
Specific behaviours	
✓ correct $\angle ACD$	
✓ correct $\angle AED$	
✓ correct justification for each angle	

Question 13

(9 marks)

Forces $\vec{b}$ and $\vec{d}$ act on a small body. Force $\vec{b}$ has magnitude 175 newtons and acts on a bearing of 307° . Force $\vec{d}$ has magnitude 115 newtons and acts on a bearing of 218° .

- (a) Sketch a labelled triangle to show the relationship between the two forces and their resultant $\vec{r}$, and show a known angle in the triangle. (2 marks)

Solution	
Angle between directions $307^\circ - 218^\circ = 89^\circ$. Angle in triangle $180^\circ - 89^\circ = 91^\circ$.	
Specific behaviours	
✓ shows resultant of sum of forces ✓ correct angle in triangle	

- (b) Determine the magnitude of $\vec{r}$ and the bearing on which it acts. (4 marks)

Solution
$ \vec{r} = \sqrt{115^2 + 175^2 - 2(115)(175)\cos 91^\circ}$ $= 211 \text{ N}$ Angle with $\vec{b}$: $\frac{\sin \theta}{115} = \frac{\sin 91^\circ}{211} \Rightarrow \theta = 33^\circ$ Bearing of resultant is $307^\circ - 33^\circ = 274^\circ$.
Specific behaviours
✓ correct expression for magnitude ✓ correct magnitude ✓ correct expression for angle ✓ correct bearing

Alternate, resolve forces:
 $\vec{b} = \langle -139.76, 105.32 \rangle$
 $\vec{d} = \langle -70.80, -90.62 \rangle$
 $\vec{r} = \langle -210.56, 14.7 \rangle$
 Mag = 211N
 Angle = 176°
 Bearing = 274°

✓ Resolve both forces
 ✓ Sum for resultant
 ✓ Magnitude
 ✓ Bearing

The work done, in joules, by a force moving a body is the scalar product of the force acting on the body, in newtons, and the displacement of the body, in metres.

- (c) Determine the work done by the resultant of $\vec{b}$ and $\vec{d}$ when it causes the body to move 214 centimetres on a bearing of 291° . (3 marks)

Solution
Angle between vectors: $\theta = 291^\circ - 274^\circ = 17^\circ$.
Work done:
$W = \vec{r} \times \vec{d} \times \cos \theta$ $= 211 \times 2.14 \times \cos 17^\circ$ $= 432 \text{ J}$
Specific behaviours
✓ correct angle between vectors ✓ correct expression for scalar product ✓ correct work done

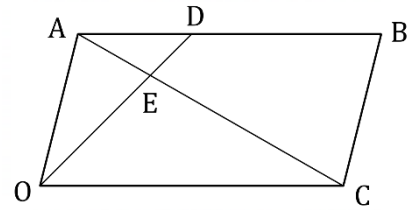
Are we accepting:
 Displacement vector = $-2\mathbf{i} + 0.77\mathbf{j}$
 Resultant vector = $-210.56\mathbf{i} + 14.72\mathbf{j}$
 Work = Displacement . Resultant vector
 = 432.5J

I think so, also if in polar form $[211, \langle 176 \rangle]$.
 $[2.14, \langle 159 \rangle] = 432$

Question 14

(8 marks)

$OABC$ is a parallelogram with $\vec{OA} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$ and $\vec{OC} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.



- (a) Determine $\vec{OB}$.

(1 mark)

Solution
$\vec{OB} = \vec{OA} + \vec{AB} = \begin{pmatrix} -5 \\ 6 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ 11 \end{pmatrix}$
Specific behaviours
✓ correct position vector

- (b) Determine $\vec{OD}$ given that D is a point on AB such that $3\vec{AD} = \vec{AB}$.

(2 marks)

Solution
$\vec{OD} = \vec{OA} + \vec{AD} = \vec{OA} + \frac{1}{3}\vec{AB} = \begin{pmatrix} -5 \\ 6 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ 23/3 \end{pmatrix}$
Specific behaviours
✓ correct expression for $\vec{AD}$ in terms of $\vec{OC}$
✓ correct position vector

OD and AC intersect at point E such that $\vec{AE} = \lambda\vec{AC}$.

- (c) Show that $\vec{OE} = (1 - \lambda)\vec{OA} + \lambda\vec{OC}$.

(2 marks)

Solution
$\begin{aligned} \vec{OE} &= \vec{OA} + \lambda\vec{AC} \\ &= \vec{OA} + \lambda(\vec{OC} - \vec{OA}) \\ &= (1 - \lambda)\vec{OA} + \lambda\vec{OC} \end{aligned}$
Specific behaviours
✓ correct expression for $\vec{AC}$ in terms of $\vec{OA}$ and $\vec{OC}$
✓ expresses $\vec{OE}$ as sum of $\vec{OA}$ and $\lambda\vec{AC}$ and simplifies

- (d) Determine the value of λ .

(3 marks)

Solution
$\vec{OE} = (1 - \lambda)\vec{OA} + \lambda\vec{OC} = (1 - \lambda)\begin{pmatrix} -5 \\ 6 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 8\lambda - 5 \\ 6 - \lambda \end{pmatrix}$
Since $\vec{OD}$ and $\vec{OE}$ are parallel then $\begin{pmatrix} -4 \\ 23/3 \end{pmatrix} = k\begin{pmatrix} 8\lambda - 5 \\ 6 - \lambda \end{pmatrix}$. Hence
$\frac{8\lambda - 5}{-4} = \frac{6 - \lambda}{23/3} \Rightarrow 23(8\lambda - 5) = -12(6 - \lambda) \Rightarrow \lambda = \frac{1}{4}$
Specific behaviours
✓ position vector for $\vec{OE}$ in terms of λ
✓ uses parallel vectors to form equation in λ
✓ correct value of λ
NB: Alternate solutions at back

Question 15

(7 marks)

Let M be the midpoint of chord AB in a circle with centre O , so that M and O are not coincident.

- (a) Let $\underline{a} = \overrightarrow{OA}$ and $\underline{b} = \overrightarrow{OB}$. Use a vector method to prove that OM is perpendicular to AB .

(4 marks)

Solution	
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \underline{b} - \underline{a}$ $\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} = \underline{a} + \frac{1}{2}(\underline{b} - \underline{a}) = \frac{1}{2}(\underline{a} + \underline{b})$ $\overrightarrow{OM} \cdot \overrightarrow{AB} = \frac{1}{2}(\underline{a} + \underline{b}) \cdot (\underline{b} - \underline{a})$ $= \frac{1}{2}(\underline{a} \cdot \underline{b} - \underline{a} \cdot \underline{a} + \underline{b} \cdot \underline{b} - \underline{b} \cdot \underline{a})$ $= \frac{1}{2}(\underline{b} ^2 - \underline{a} ^2)$ $= 0 \text{ since } \underline{a} = \underline{b} = \text{radius of circle}$ When the scalar product of two non-zero vectors is zero then vectors must be perpendicular and so OM is perpendicular to	
Specific behaviours	
✓ vector expressions for $\overrightarrow{AB}$ and $\overrightarrow{OM}$ ✓ forms and expands scalar product ✓ simplifies, explaining why $\underline{a} = \underline{b}$ ✓ completes proof using result of scalar product	

Are we happy to award the last mark if they just conclude by saying "therefore perpendicular" or do they need this complete explanation?

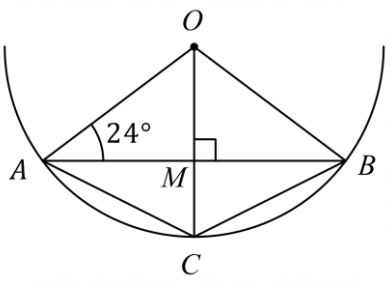
Gin: Ok with "therefore perpendicular", but happy to go with what the team decides

If use the wrong vectors:
Lose the first mark
Award the rest if proof correct
Deduct 3rd mark for false proof

OM is extended to cut the minor arc AB at C and $\angle OAM = 24^\circ$.

- (b) Determine, with justification, the size of $\angle CBM$.

(3 marks)

Solution	
	$\angle AOM = 90^\circ - 24^\circ = 66^\circ$ (Angles in right triangle)
	$\angle CBM = \angle AOC \div 2 = 66^\circ \div 2 = 33^\circ$ (Angles at centre and circumference)
Specific behaviours	
✓ sketch diagram - may be seen in part (a) with annotations from this part ✓ deduces $\angle AOM$, with justification ✓ correct angle, with justification	

Question 16

(8 marks)

- (a) Players in a club are assigned to the junior or senior squad, but not both. Determine the number of distinct ways in which a coach can select 12 players when 3 must be chosen from the junior squad of 8, and the remainder from the senior squad of 11. (2 marks)

Solution
Number of ways is $\binom{8}{3} \binom{11}{9} = 3080$
Specific behaviours
✓ one correct combination
✓ correct number of ways

- (b) As part of a training exercise, a squad of 16 players must be divided into three groups, one with 7 players, one with 5 players and the other with 4 players. Determine the number of distinct ways in which the division can be made. (2 marks)

Solution
Number of ways is $\binom{16}{7} \binom{9}{5} = 1\,441\,440$
<i>NB: 5 alternative nCr pairs exist to obtain number of ways</i>
Specific behaviours
✓ one correct combination
✓ correct number of ways

- (c) When r distinct objects are divided into groups of x, y and z objects, where $x + y + z = r$, prove that $\binom{r}{x} \binom{y+z}{z} = \binom{r}{z} \binom{y+x}{x}$. (4 marks)

Solution and alternative	
LHS: The first term is number of ways to choose a group of x objects from r objects. The second term is the number of ways to choose the next group of z objects from the remaining $y + z$ objects, and then the objects left over make up the last group. The RHS expression is the same, except that the group of z objects is chosen first and then the group of x objects is chosen next. The expressions are equal because regardless of the order in which the groups are picked, the number of distinct ways to choose the groups does not change.	Note that $r - x = y + z$ and $r - z = y + x$. $\begin{aligned} \text{LHS} &= \binom{r}{x} \binom{y+z}{z} \\ &= \frac{r!}{x! (r-x)!} \times \frac{(y+z)!}{z! y!} \\ &= \frac{r!}{x! z! y!} \\ &= \frac{r!}{z! (r-z)!} \times \frac{(y+x)!}{x! y!} \\ &= \binom{r}{z} \binom{y+x}{x} \\ &= \text{RHS} \end{aligned}$
Specific behaviours	
✓ explains first term of LHS or RHS of expression ✓ explains second term of same expression ✓ explains other side of expression ✓ explains why expressions are equal	✓ expands one side into factorials ✓ notes equivalences

Are we penalising for false proofs?
Proposing: -1 for false proof (4th mark)

Agreed.
 Accept "inelegant proof" i.e. working both sides to a common point like the 3rd line or working?

Question 17

UNITS QUESTION (part c)

(9 marks)

A small model boat leaves buoy A with constant velocity $\begin{pmatrix} 4 \\ -31 \end{pmatrix}$ centimetres per second, and t seconds later it reaches buoy B . Buoys A and B are moored in a river and have position vectors $\begin{pmatrix} 31 \\ 455 \end{pmatrix}$ and $\begin{pmatrix} 127 \\ -217 \end{pmatrix}$ centimetres respectively.

A steady current is flowing in the river with velocity $\begin{pmatrix} m \\ n \end{pmatrix}$ centimetres per second.

- (a) Show that the constants m and n are related by the equation $n = 3 - 7m$. (4 marks)

Solution	
$\overrightarrow{AB} = \begin{pmatrix} 127 \\ -217 \end{pmatrix} - \begin{pmatrix} 31 \\ 455 \end{pmatrix} = \begin{pmatrix} 96 \\ -672 \end{pmatrix}$	
Let $\vec{v}$ be the velocity of the boat over ground: $\vec{v} = \begin{pmatrix} 4 \\ -31 \end{pmatrix} + \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} m+4 \\ n-31 \end{pmatrix}$	
Then $t\vec{v} = \overrightarrow{AB}$ and so	
$t = \frac{96}{m+4} = \frac{-672}{n-31}$ $n-31 = \frac{-672}{96}(m+4)$ $n = 3 - 7m$	
Specific behaviours	
✓ correct displacement vector ✓ correct velocity vector ✓ forms equation with reasoning ✓ simplifies into required form	

For second mark, accept:

$$\begin{pmatrix} 4 \\ -31 \end{pmatrix} + \begin{pmatrix} m \\ n \end{pmatrix} = \lambda \begin{pmatrix} 96 \\ -672 \end{pmatrix}$$

OR

$$\begin{pmatrix} 4 \\ -31 \end{pmatrix} t + \begin{pmatrix} m \\ n \end{pmatrix} t = \begin{pmatrix} 96 \\ -672 \end{pmatrix}$$

- (b) Given that the speed of the current is $\sqrt{17}$ centimetres per second and that $n > m$, determine the velocity vector of the current. (3 marks)

Solution	
From speed, $m^2 + n^2 = 17$, and from above $n = 3 - 7m$.	
Solving these simultaneously gives: $\{m = 1, n = -4\}, \{m = -0.16, n = 4.12\}$.	
Since $n > m$ then the velocity of the current is $\begin{pmatrix} -0.16 \\ 4.12 \end{pmatrix}$ centimetres per second.	
Specific behaviours	
✓ forms equation using speed ✓ solves simultaneously to obtain two solution sets ✓ states correct velocity vector for current	

$$m = -\frac{4}{25}$$

$$n = \frac{103}{25}$$

- (c) Determine how long the boat takes to travel from A to B . (2 marks)

Solution	
$t = \frac{96}{m+4} = \frac{96}{-0.16+4} = 25 \text{ seconds.}$	
Specific behaviours	
✓ correct time ✓ includes correct units	

Question 18

(7 marks)

- (a) Passwords are to be formed using all 8 symbols in the set $\{*,!, =, +, ^, \%, \#, \&\}$ without repetition, such as $= !\% + \&^{\#} *$. Braces $\{\}$, full stops and commas are not classified as symbols here. Determine how many different passwords can be made

- (i) with no other restrictions.

(1 mark)

Solution
$8! = 40\,320$ passwords can be formed.
Specific behaviours
✓ correct number

- (ii) that start with # or end with %.

(3 marks)

Solution
Start with #: $1 \times 7! = 5040$
End with %: $1 \times 7! = 5040$
Start with # and end with %: $1 \times 1 \times 6! = 720$
Using inclusion-exclusion principle, the number of passwords that start with # or end with % will be $2 \times 5040 - 720 = 9360$.
Specific behaviours
✓ indicates correct number that start/end with required character
✓ indicates correct number that start and end with required characters
✓ correct number of passwords

Three-digit numbers are to be formed using permutations of three digits from the set $\{3, 4, 5, 6, 7, 8, 9\}$ without repetition. Determine how many even numbers less than 680 can be formed in this way.

- (b)

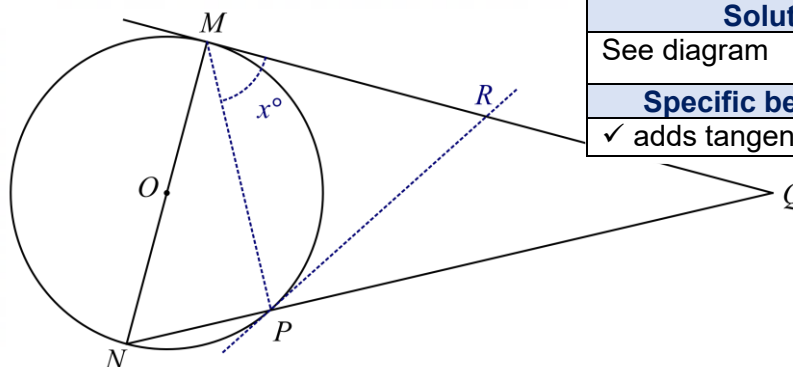
(3 marks)

Solution
In the following cases we count ways to pick the first, then the last and then the middle digit:
6 - even - even: $1 \times 1 \times 1 = 1$
6 - odd - even: $1 \times 3 \times 2 = 6$
4 - any - even: $1 \times 5 \times 2 = 10$
3 or 5 - any - even: $2 \times 5 \times 3 = 30$
There are $30 + 10 + 7 = 47$ even numbers.
Specific behaviours
✓ indicates attempt to split into mutually exclusive cases
✓ correctly counts number for at least 2 cases
✓ correct number

Question 19

(8 marks)

In the diagram, MQ is tangential to the circle with centre O and diameter MN at M . Line NQ cuts the circle at P .



Solution
See diagram
Specific behaviours
✓ adds tangent and chord

- (a) The tangent to the circle at P cuts MQ at R . Show this on the diagram, draw chord MP and let $\angle PMR = x^\circ$. (1 mark)
- (b) Prove that $\angle RPQ = 90^\circ - x^\circ$. (4 marks)

Solution
$\angle MNP = \angle PMR = x^\circ$ (alternate segment)
$\angle MPR = \angle MNP = x^\circ$ (alternate segment)
$\angle MPN = 90^\circ$ (angle in semicircle)
$\angle RPQ + \angle MPR + \angle MPN = 180^\circ$ (straight angle)
Hence $\angle RPQ = 180^\circ - 90^\circ - x^\circ = 90^\circ - x^\circ$
Specific behaviours
✓ deduces that $\angle MNP = x^\circ$
✓ deduces that $\angle MPR = x^\circ$
✓ justifies why $\angle MPN = 90^\circ$
✓ completes proof
NB: One alternate solution at back

Many variations, accept any rigorous and logical proof.

- (c) Prove that $RM = RQ$. (3 marks)

Solution
$\triangle MRP$ is isosceles since from above $\angle PMR = \angle MPR = x^\circ$ and so $RM = RP$.
In $\triangle MPQ$, $\angle MPQ = 90^\circ$ (see above) and so $\angle MQP = 90^\circ - x^\circ$ (angle sum in triangle).
Hence $\triangle QRP$ is isosceles since $\angle RPQ = \angle RQP = 90^\circ - x^\circ$ and so $RP = RQ$, but from above $RM = RP$ and so $RM = RQ$.
Specific behaviours
✓ deduces $RM = RP$
✓ deduces $\angle RQP = 90^\circ - x^\circ$
✓ deduces $RP = RQ$ and completes proof

Solution
$\angle OEC = \angle DEA$ $\angle ECO = \angle EAD$ $\therefore \triangle OCE \sim \triangle DAE$ $\overrightarrow{AD} = \frac{1}{3} \overrightarrow{OC}$ $\overrightarrow{DE} = \frac{1}{3} \overrightarrow{OE}$ $\overrightarrow{AE} = \frac{1}{3} \overrightarrow{EC}$ $\overrightarrow{AE} = \frac{1}{4} \overrightarrow{AC}$ $\lambda = \frac{1}{4}$
Specific behaviours
✓ proves AA triangle similarity ✓ recognises scale factor of a third ✓ correct value of λ

Solution
$\angle MNP = \angle PMR = x^\circ$ (alternate segment) $\angle MPN = 90^\circ$ (angle in semicircle) $\angle PMN = 90^\circ - x$ (angle sum of a triangle) $\angle RPQ = \angle NPS$ (vertically opposite) $\angle NPS = \angle PMN = 90 - x$ (alternate segment) Hence $\angle RPQ = 90^\circ - x^\circ$
Specific behaviours
✓ deduces that $\angle MNP = x^\circ$ ✓ deduces that $\angle PMN = 90 - x^\circ$ ✓ justifies why $\angle RPQ = \angle NPS$ ✓ completes proof